Number Games:

Pandigital Numbers, Friedman Numbers, and e

Richard Wong

SMMG 2020

Slides and worksheets can be found at
http://www.ma.utexas.edu/users/richard.wong/Notes.html

Pandigital Numbers

Definition

A **pandigital number** is an integer that uses each digit 0-9 **exactly once** in the significant digits of its decimal representation.

Example

- 1234567890 is a pandigital number.
- 0123456789 is not.
- 11234567890 is not a pandigital number, but it is a pandigital number with redundant digits.

Pandigital Numbers

Definition

A **pandigital number with redundant digits** is an integer that uses each digit 0-9 **at least once** in the significant digits of its decimal representation.

Definition

A **pandigital number** is an integer that uses each digit 0-9 **exactly once** in the significant digits of its decimal representation.

Work on the Pandigital numbers section of the worksheet!

Friedman numbers

Definition

A **Friedman number** is an integer that can be non-trivially expressed as a formula using each of its significant digits exactly once, along with the operations $(+, -, \times, \div)$, additive inverses, parentheses, and exponentiation.

Example

- (n) is a trivial way to express an integer n. So that means that the single digit numbers cannot be Friedman numbers.
- 343 is a Friedman number, since $(3+4)^3 = 343$.

Work on the Friedman numbers section of the worksheet!

Euler's constant

So far today we have been investigating numbers and their decimal representations.

However, we will now investigate a number that is **irrational**, and even **transcendental**.

$$e = 2.718281828459045\ldots$$

However, we don't need to know the significant digits of a number to define it.

$$\sum_{k=0}^{\infty} \frac{1}{k!} = e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

We can use these formulas (among others) to approximate the digits of e.

Work on the Approximating e section of the worksheet!

How closely do you think we can approximate e using a pandigital formula,

using the operations $(+, -, \times, \div)$, additive inverses, parentheses, and exponentiation?

A pandigital formula

$$e \approx (1 + 9^{-4^{6\times 7}})^{3^{2^{85}}} + 0$$

This formula was found by Richard Sabey in 2004, and is correct to 18×10^{24} digits.

That's 18 trillion *trillion* digits!

The inspiration

$$e \approx \left(1 + 9^{-4^{6\times 7}}\right)^{3^{2^{85}}} + 0$$

- First recall that $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$.
- Now note that $3^{2^{85}} = 9^{2^{84}} = 9^{4^{42}} 9^{4^{6\times 7}}$.
- Then, set $n = 3^{2^{85}} \approx 1.846 \times 10^{25}$.

The error term

To approximately determine the error term, we need some analysis.

- Note that $\left(1+\frac{1}{n}\right)^n < e < \left(1+\frac{1}{n}\right)^{n+1}$.
- Therefore,

$$\begin{aligned} e - \left(1+\frac{1}{n}\right)^n &< \left(1+\frac{1}{n}\right)^{n+1} - \left(1+\frac{1}{n}\right)^n \\ &= \left(1+\frac{1}{n}\right)^n \left(\frac{1}{n}\right) \\ &= \frac{e}{n} \end{aligned}$$

- Hence, given a choice of n, the error term is less than $\frac{e}{n}$.